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Musical composition – an ineffable act between fantasy and arithmetical and geometrical rigor

Résumé

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Motto: *"We can reduce all to numbers, including Beethoven's music. But we do not hear numbers, we hear music".* Pierre Schaeffer

Chapter 1

Link between music and mathematical sciences – always present and proved since the Antiquity

The topic of the relation between music (the art of sounds) and exact sciences is more and more complex in the light of the achievement of 20^{th} century music.

Today, the new computer technique and the electronic devices are intensely present both in the composition act and in the interpretation and the understanding of the works of art.

Until reaching this level, music had always relations with the sciences, which were present at an organic level.

The perception of the contacts between the two domains, with their positions, can only enrich the act of creation. A decline in this act is present when the inspiration is missing, when fantasy and lyricism are incomplete and when there intervene (out of snobbism) a search for new laws in composition, other than the ones specific to the art, whose final goal is the esthetic category of the beautiful.

The relation music-mathematical sciences was present as early as the Antiquity and emphasized by a large number of Greek authors: Pythagoras of Samos (centuries 4-5 BC), Aristoxenos of Tarentum (4th century BC), Plato (centuries 4-5 BC), Aristotle (4th century BC), Vitruvius Pollio (1st century BC).

During the Middle Age, the art of sounds was considered an important subject in education, next to the other sciences of the Quadrivium: arithmetic, geometry, music and astronomy. Severinus Boethius (475-524 AC), Leonardo Fibonacci (centuries 12-13), Gioseffo Zarlino (1517-1590), René Descartes (Cartesius) (1596-1650), Jean Philippe Rameau (1683-1764), Herman con Helmholz (1821-1892) are some of the significant authors for Middle Age, Renaissance and Modern times, who conducted research on the everlasting connection between music and mathematics, as it was know from the philosophy and from the artistic output.

Gottfried Wilhelm Leibniz (1646-1716) wrote: "music is an exercise of covert arithmetic and the one who devotes himself to it is not aware of the fact that he is handling numbers".

From the above-mentioned authors I kept the following assertions, which I considered the most significant:

- a) From Pythagoras the Pythagorean concept on the acoustics and the theory of figured numbers (triangular numbers, square and rectangle numbers, applicable in modal structures)
- b) From Plato this discoveries on "the golden ratio" and on the heterophonic mode (this acoustic mode is constructed on the proportions of the numbers 2, 3, 4, 9, 8, 27 and represents the harmony of the universe)
- c) From Aristotle the philosophical implications on the relation musicmathematics (through the classification he proposes inside the sciences: theoretical sciences, practical sciences and poetical sciences). Music, together with poetry and architecture is in the poetical sciences group.
- d) Aristoxenos of Tarentum (considered the greatest musician of Antiquity) was also interested in the relation between the art of sounds and mathematics in his studies on acoustics, researching micro-intervals.
- e) From Vitruvius Pollio I took the idea of symmetry of the form, his theory of the tetrachords (diatonic, chromatic and enharmonic tetrachords), the relation between theatrical acoustical ceramic and the tetrachords systems.
- f) Leonardo Fibonacci left mankind an interesting arithmetic equivalent of the "golden ratio" in the law of organic growth and the law of Fibonacci series.
- g) Gioseffo Zarlino, important theoretician from the Renaissance, was preoccupied by the acoustic non-tempered standards and by the definition of the major and minor scales.

- h) Acoustical studies were made by René Descartes (Cartesius) too, who looked on music aesthetics in concordance with acoustics and music psychology.
- i) Jean Philippe Rameau, an important theoretician, was considered the founder of the classical harmony concepts
- j) The relation between music and mathematical sciences is studied by Hermann von Helmholz too, who focused on the theory of physiology and acoustics of music; in his opinion the musical "harmony" (the consonances) creates continuous excitations and the dissonances discontinue excitations.

The evolution of this strong connection between music and science and its particularity according to historical periods is the result of continuous research, which in the 20th century developed into several aspects connected to the investigation of the artistic conception act, to the understanding of the works of art, to modern analytic methods.

In the 20th century there are some artistic styles which emphasize this connection (perhaps, some times, not in the favor of the lyric expression of the artistic message): 12 tone music, serial music, atonal music, stochastic music, random music, minimal music, repetitive music etc.

Instruments of playing music developed very much, accordingly to the technical and scientific evolution, next to the traditional sources being used electronic devices and computers.

From the aesthetics point of view, semiotic and semantic theories determined the development of quite an industry of explanations of the role of graphic sign and of the sound signal, explanations that are interesting to some point, but which cannot get inside the profound mechanism of the artistic creation and cannot establish precise rules of how to compose.

The relation music-exact sciences must not have a forced effect, coming from outside the human creativity. In this line, I express some reserve towards those styles and those composers which have no talent to create a lyric expression and which transforms the art of sounds into a master craft of mixing uninteresting frequencies and rhythmic impulses, without coming to something expressive and clearly constructed. I see expressiveness and the national character of the music as being two essential and eternal coordinates of any work of art, regardless the historic moment when it was created.

In my opinion, the presence of rigor in the construction of music is capable on generating beautiful music even in this century, when technology competes with the sentiment; so the rigor of construction is not a characteristic of the ugly and anti-human music only.

In fact, from another angle, my interest is directed towards a plea for the presence of the consonance in music, this being not an out fashioned concept, but an essential way of expressing the harmony, the beautiful, the light, the most beautiful arithmetic relations which human intelligence can produce, of expressing the human soul and spirit.

Chapter 2

The magical squares and their presence in music

A. Definition of the magical square

We call "magical square" a square of numbers were there are n^2 numbers, aligned consecutively or not, so the sum of the numbers place on each of the two diagonal lines to be equal with the sum of the numbers on each column or row. This **constant sum** is named "the magical number" of the square.

B. History of the magic square

The astrologists of the Antiquity, for instance the Chinese ones, in the 7th century BC, then the Arab ones, were building talismans to which they were giving magical powers. These squares were popular in Europe in the Renaissance – for instance Albrecht Dürer, in his painting The Melancholy, painted a magic square with the magic number 34. In the same period (14th century) the Greek mathematician, Manuel Moscopoulos wrote about the magical squares, which he named arithmetic squares ("tetragonon arithmon"). He is first to present a general method of building a square of odd squares and double even squares.

The issue of magical squares, seen as arithmetic fun, became a delight during times, being in the attention of great mathematicians like Euler or like Benjamin Franklin.

Magical squares represent an attractive domain until our days, without the magical attribute, of course.

C. Rules of making magical squares

The odd magical squares may be made following the method of Bachet de Méziriac, who published in 1612 the book "Nice and delightful problems which may are solved by numbers", or following the method of Philippe de la Hire (1700).

D. Applications in music of the odd magical squares

Those arithmetic squares can be used in music, their numeral equivalence creating very interesting modal systems, frequently used by composers, during the centuries.

If we connect an interval to each figure, in ascendant order, starting with the semitone, we can trace the following parallelism:

1	=	Minor 2 nd	6	=	Augmented 4 th
2	=	Major 2 nd	7	=	Perfect 5 th
3	=	Minor 3 rd	8	=	Minor 6 th
4	=	Major 3 rd	9	=	Major 6 th
5	=	Perfect 4 th	1 0	=	Minor 7 th
			11	=	Major 7 th

In this chapter I presented a demonstration of modes applicable to the arithmetic squares of the 3,5,7,9 order and to the square named "hypermagical square", from which results constant rules of forming chords:

- The reading of the figures in rows gets the intervallic sum of the 2 diagonal lines.
- The reading of the figures in columns gets the result of the substraction of the 2 diagonal lines
- Both the chords in rows and in lines are disposed symmetrically

- There is a symmetry also in the plan of the modal structures
- Each arithmetic square presents 2 intervallic constants of the 2 diagonal lines, by which the whole modal system may be built, which will have like symmetrical axes the numeric correspondence given by the "magical sum" of it
- The system may be repeated indefinitely and is applicable also to arithmetic squares made of random figures

Making a numeric equivalence in the plan of durations too, there will often appear rhythmic imitations structures, both when reading rows and columns.

Example:

The square of the $5^{\rm th}$ order used for rhythmic structures, with basic value the $16^{\rm th}$ note

3	16	9	2 2	15
2 0	8	21	14	2
7	2 5	13	1	19
2 4	12	5	1 8	6
11	4	17	1 0	2 3

Horizontal reading (rows)







Chapter 3

The relation between musical forms and geometric forms

"In art everything is made up after three fundamental geometrical forms: sphere, cube, and cylinder. You have to learn to draw those very simple forms, and afterwards you can draw whatever you want."

Paul Cézanne

The relation music-mathematical sciences was present forever and proved since the Antiquity not only in the acoustics but also in the field of musical architecture.

Making musical forms often has to do with geometrical forms. It was not incidentally said that music is an architecture with evolves in time.

This relation may be seen and proved from various angles, which seems to be fundamental during the ages.

A. The relation between the five solids of Plato and the musical form

The five convex regular solids (three-dimensional geometrical shapes) are: the tetrahedron, the hexahedron (the cube), the octahedron, the dodecahedron and the icosahedron.

Which can be the link between the shape of a musical construction (form) and those Plato solids?

It exists organically, can be proven and one can show the close connection between the space geometry thinking and the music creation thinking.

It is known the fact that in the art of sounds there are few fundamental construction schemes, which generated among centuries various specific forms and genres like

A (single-part form)

AB (binary form)

ABA (the ternary form)

AAB or ABB (bar-form)

ABACABA (rondo-form)

If we associate the above-mentioned regular tetrahedrons the reading of a mathematic Hamiltonian circuit, we will have the revelation of discovering the presence of the music construction schemes used in all styles. The basic circuit imagined by Hamilton on a regular polyhedron is the one which forms a closed circuit, along the edges, passing only once through each peak.

- 1. If this circuit is made along the edges of a hexahedron (cube), having the coordinates A, B and C, the road to be taken is ABACABA, which is the classical rondo-form.
- 2. Following the same principle, the relation between a tetrahedron and the musical architecture generates the scheme ABA or AAB (bar-form)
- 3. Reading by the same rules an octahedron develops the following musical pattern: ABBBA or ABCBA.
- 4. The relation between the musical form and the dodecahedron (considered the measure of the Universe) is a unit: A.
- 5. The icosahedron, consisting of 20 equilateral triangles will suggest a mono-part form also, replicable to infinity: A.

Concluding, we may observe that:

- a) the octahedron and the hexahedron are 2 Plato solids which generates musical forms of the same family (ABCBA or ABACABA)
- b) the icosahedron and the dodecahedron gives the same musical pattern (A)
- c) the tetrahedron has a unique form (AAB or ABB)

B. The relation of symmetry present in sounds architecture

Symmetry is an essential coordinate in composition and a rigorous geometric concept. Both in geometry and music there are several types of symmetry:

- 1) Bilateral symmetry (plane symmetry) which exists in music in palindrome canons in which the melody which imitates is the recurrence of the first melody. Many composers, among them Joseph Haydn, Johann Sebastian Bach, Ludwig van Beethoven, Paul Hindemith, Arnold Schönberg, Alban Berg etc., used this kind of schemes, for obtaining effects in counterpoint.
- 2) Translational symmetry. This can be of two kinds:
 - a) rhythmical translation symmetry
 - b) cylindrical translation symmetry

Both types are used in music, in various composition processes. For instance, rhythmical translational symmetry (of infinite ratio) can be frequently found in all repetitive music, and cylindrical translational symmetry (of finite ratio) is present in the typology model – sequence – cadenza.

- 3) Plane rotating symmetry. This can be of two kinds:
 - a) rotating cyclic symmetry (rotation without reflection)
 - b) rotating dihedral symmetry (with reflection)

The rotating cyclic symmetry has equivalence in musical composition in the articulation of the architecture, a characteristic example being the exposition of the fugue.

The musical equivalent of the dihedral rotation may be considered the palindrome type structures, transposed n times.

- 4) Rotating symmetry in space. This can be of three kinds:
 - a) symmetry made of a rotation in plane and a orthogonal translation in it
 - b) symmetry made of a rotation in plane accompanied by dilatation
 - c) symmetry made of a rotation, a translation and a dilatation

All those kinds of rotating symmetries are frequently used in the musical creation process. The Theme and variations form is clearly belonging to the typology of rotating symmetry with translation and dilatation in space, because the periodic replay of an initial theme asks for a cyclic re-evaluation, and its more and more ornamented shape suggests a dilatation of this symmetry.

C. Structural rhythm – dynamic symmetry – logarithmic spiral

In musical scores, rhythm is present both as a succession of equal or different values and as a succession of structures, which ends uniformly or asymmetrical in the fields of an architectural configuration.

Musical forms, in their articulation, both at the micro-structural and macrostructural levels presents a some interior symmetries, which by their continuous projection during the piece suggests the existence of a logarithmical spiral.

The structural cadenzas of the musical language may be grouped on binary or ternary periodicity bases, which are present both at the level of primary configurations and at the level of large surfaces. Also, the archetype of construction model-sequence-cadenza is also a translation of the logarithmical spiral from geometry to the art of sounds.

The structural binary and ternary rhythms, present in music, generated some typologies of architectural organic articulation , like the relation verse-rhythm established as early as in the Antiquity two large groups of rhythms: dissyllabic and trisyllabic.

So, the structural binary rhythms may have their roots in the antic rhythms of spondees (θ θ) and pyrrhic (ε ε), and the structural ternary rhythms in the antic rhythms of the dactyls (θ $\varepsilon\varepsilon$), anapests ($\varepsilon\varepsilon$ θ) and amphibrach (ε θ ε). Also, the rhythms molos (θ θ θ) and tribrachic ($\varepsilon \varepsilon$ ε) may be found in music, but their uniformity converge more towards static cadenzas.

The rhythm anapest ($\epsilon\epsilon \theta$) presents 2 short values and a third long value. This sequence is identical to the architectonic form AAB which I mentioned before, regarding the relation between Plato solids and the musical architecture and regarding the typology model-sequence-cadenza.

Cornel Ailincăi, in his book "Introduction in the grammar of the visual language" concludes: "If we think of rhythm in the most general meaning, its existence requires a continuous, uninterrupted movement of extension (the

Greek word **rhythmos** comes from **reo** – to flow), but in the same time requires a division of the infinite extension in periods to be repeated by a rule. So, rhythm is different of ordinary movement by a succession of phases whose discontinuity ensures the reconstruction of the movement".

Chapter 4

The relation between the mosaic and musical architecture

Geometrically the mosaic is built up of pre-defined elements, like squares, rhombs or triangles. Inside the mosaic one can make original shapes, using various combinations of the above mentioned elements.

Mosaic was frequently use since Antiquity, in the decorative arts, using its fundamental elements which made an homogeneous field paved with squares, equilateral triangles and hexagons.

The Romans, the Persians, the Chinese, the Japanese were masters or ornaments and they made all kinds of mosaic surfaces, using repeated elements.

The mosaic may be used in music architecture, at the micro-structural level, as well as at the macro-structural one.

It can be found, for instance, in the technique of collage where music entities apparently dissimilar may give together a nice ensemble impression. We can take Gustav Mahler's symphonies as a characteristic example for this.

Also, we can easily disclose mosaic in the juxtaposing of micro-forms, differently repeated in two dimensions. For this, the typical example is the passacaglia form. Here we meet the phenomenon of figure – background, where the perception discovers two simultaneous aspects of the same image. One is the background, the rhythm and the melody of the passacaglia, which remains unchanged, which forms the homogeneous pavement, and the other one is the figure, built up of different polyphonic variations, laid over the background.

The ostinato form (which may found its equivalent in the mosaic decorations) is frequently found in music. Among the famous examples there are the $32^{nd} C$ minor Beethoven variations, the 4^{th} Hindemith string quartet op. 32 (last movement), the 3^{rd} Bartok quartet (first movement), J. S. Bach's *Crucifixus* from the D minor Mass, D. Buxtehude's E minor *Ciaccona* for solo organ, M.

Reger's *Introduction, Passacaglia and Fugue* for 2 pianos op. 96, J. Brahms' *Variations on a Haydn theme* op 56a (last movement) and his 4th Symphony (last movement), I. Strawinski's *Psalms symphony*, A. Webern's *Passacaglia op. 1* for orchestra, A. Honegger's *Pacific 231* etc.

Chapter 5

A. The notation of the "parlando rubato" sections used in my own compositions

Around 1977-78, when I elaborated *Incantations I* and *II*, inspired of manuscripts of Filothei Sin Agăi Jipei, which I analyzed, focusing on their rhythmic and temporal aspect, I was more and more preoccupied of finding a most rigorous way of writing what we call "parlando rubato".

It may seem strange to want to write exactly a "parlando rubato" music, but I wanted to create a type of music consisting of rhythmical configurations at the micro-structural level, to be used by each of the interpreters, so that each concert variant to be as closed to my psychological tempo and, especially, to the refined waves I imagined in the creational process.

I used the first attempts of this kind in the cycle *Incantations* and after I introduced that writing with consistency in all slow tempo music that followed $(3^{rd}, 4^{th}, 5^{th} \text{ and } 6^{th} Symphonies$, the *Concerto for flute, viola and chamber orchestra*, opera *The Snow Queen*, ballet *The Little Mermaid*, the *Concerto for strings* etc.), this writing being nowadays a constant method of imagining rhythmic rows for various texts or polyphonic layers (some technique may be applied also in homophonic music).

Starting from the well-known Fibonacci series (1, 2, 3, 5, 8, 13, 21 etc.), including its translations (like 1, 3, 4, 7, 11, 18, 29, 47 or 0, 2, 2, 4, 6, 10, 16, 26, 42, 60 etc. or 0, 3, 3, 6, 9, 15, 24, 39 etc. or 1, 4, 5, 9, 14, 23, 37) I started to imagine various non-demoting series applied to a time unit (for example the quarter note) and to its equal divisions in 2, 3, 4, 5, 6.

Those rhythmic structures resulting of the using of non-demoting abome mentioned partitions presents some constant characteristics:

1. Each configuration has a certain phrasing, both at the micro-structural and macro-structural levels, given by the sum of the terms used in those series

2. The usage of simultaneous series (Fibonacci's series translations) may give the same pattern on the vertical plane (regardless the subdivision on the main unit), if sum of the terms is the same

Example: the superposing of the series 3, 2, 3, 1, 3, 2, 3, 1 ... and 1, 3, 1, 4, 1, 3, 14, ...

3+2+3+1=9 1+3+1+4=9

- 3. It becomes clear that by applying Fibonacci series to 5, 6, 7 etc. equal impulses subdivisions of the main value the result is rhythmic structures with refined waving, which are not always very strict (especially if they are used in combinations, like quintuplet and sextuplet); but if they are written in this technique, they may become a pattern to which each interpretation is close, with no major difference from the initial structure had in mind by the composer.
- 4. The Fibonacci series is the most pure type of the two-beat accumulational series, but the infinite reproduction of the initial series is the Pascal triangle, the richest diagram in algebric and geometric properties. This diagram includes figurative numbers series triangular, tetrahedric, pentagonal etc., often present in the rhythmic configuration of musical pieces.

B. The notation of some rhythmic structures based on the accumulational series of triangular numbers.

The series of the triangular numbers is 1, 3, 6, 10, 15, 21, 36, 45 etc. From their various combinations (taken in row or by one, two or several leaps), by their application to various basic time units or to subdivisions of this units, by their use in various partitions of non-demoting series one can creat endless rhythmic combinations.

This rhythmic structures has always the same invariable, which is the sum of the terms used in the numeric loops gives always the periodicity of the rhythmic figures at the micro-structural or macro-structural level. This phenomenon may be the explanation of the frequent assertion of the musicologists that "micro-structure generates macro-structure".

Example:

The periodicity is of 20 impulses (1+3+6+10=20)



Example of notation of rhythmic structures based on Fibonacci series numbers:

Liana Alexandra – 3rd Symphony, Editura muzicala, 1985, 2nd movement, page 69.

Chapter 6

Epilogue

The complex relations between the art of sounds and mathematic scienes which I tried to investigate in this study and to demonstrate in musical examples, with which creative thinking operates frequently, may define, by a certain point, the rational aspect of the artistic creation. But besides this investigations and demonstrations must be always the inspiration, the ineffable aspect of human creativity, which offers always connexions and original ways of expressing the beautiful in art.

Together with the intellect, the inspiration is the one which gives aesthetic value to a work of art. When inspiration is not present, one may produce abstract schemes, inventive charts, and scholarly written explanations, but lacking that warmth specific to any creative act.

In order to emphasize more these assertions, I cited in the final part of my study some wisdom words written by great people in the history of mankind, which stress the aesthetic dimension of the work of art, without which no final product of human fantasy, harmoniously knitted with brainpower, may exist. This words were written by Democritus, Heracles, Simylus, Plato, Pascal, Michelangelo, Shakespeare, J. J. Rousseau, Nicolae Iorga, Barbu Ştefănescu Delavrancea, Tudor Vianu, Arthur Honegger, Anton Webern, George Enescu, Abraham Moles, P. A. Michelis and Confucius.

For this résumé I have chosen fragments from the ideas of Arthur Honegger, George Enescu and Confucius.

Arthur Honegger:

"Writing music is like putting a ladder without fixing it. With no scaffolding, a building in construction may stay in place only by a miracle, a miracle of the internal logic, of an inner sense of proportion. I am in the same time the architect and the spectator of my works; I work and I analyze my work..."

George Enescu:

"It is true that music is related with mathematics. But the great composers were no mathematicians; or, if you like better, they were, but in an unconscious way. Bach, with his genius, sensed the superior connection between the fragments of his works. His pieces may disclose mathematical ratio and proportions, but Bach himself has not created them by logical, deductive thinking. The composer is a mathematician, or more precisely, the mathematical spirit dominates him like the profound intelligence".

Confucius:

"If you want to know if a country is well governed, you have only to listen to its music."

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